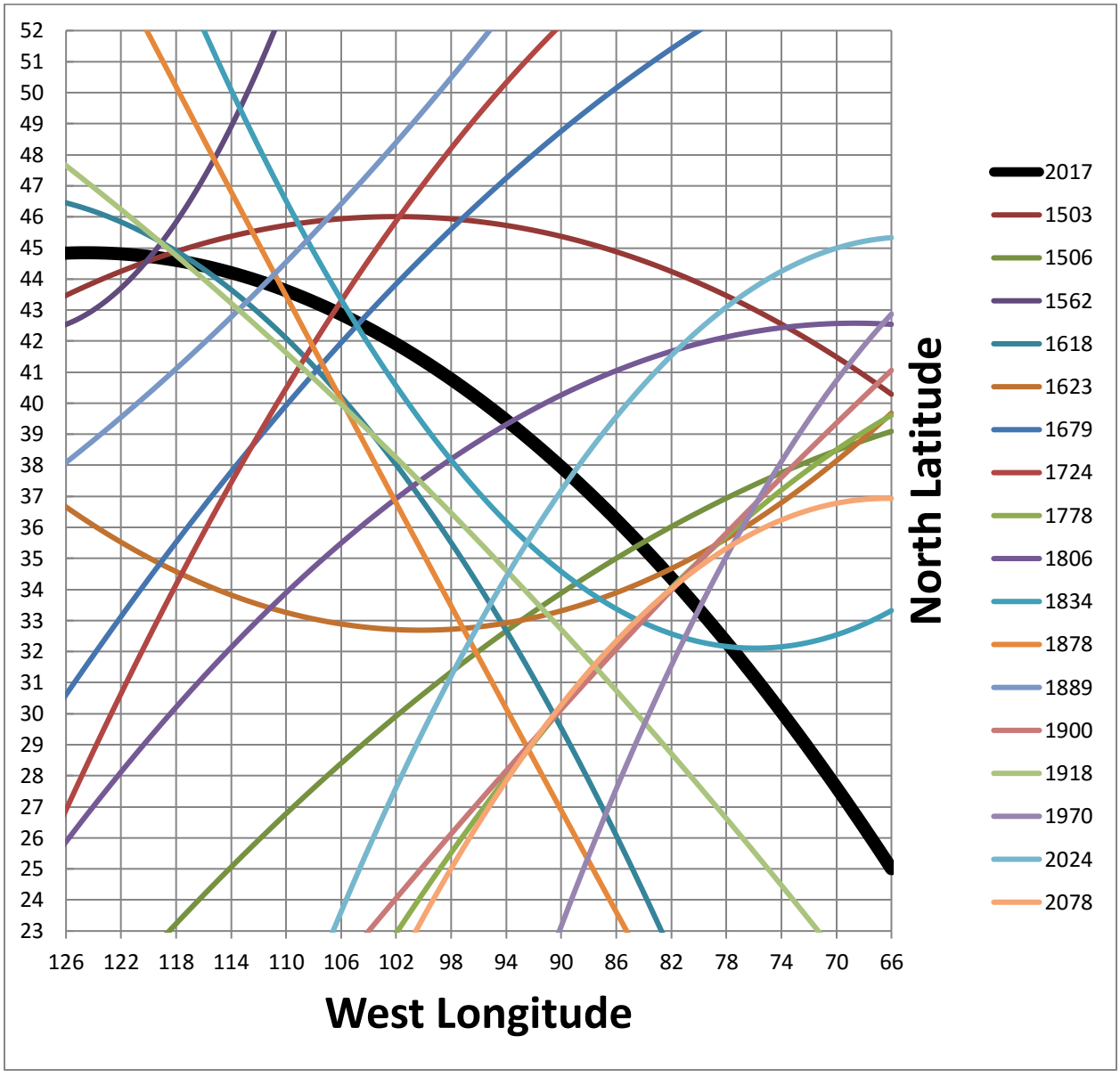
X Marks the Spot – Quadratic Equations

One interesting feature of the August 21, 2017 total solar eclipse across the continental United States is that it crosses the path of other, older eclipses that have occurred since 1503 when North America was first discovered. Here are what the eclipse paths look like for previous eclipses. The black track is for August 21, 2017.



Fun with Quadratic Equations!

For the mathematically-inclined, this table gives a quadratic equation that represents the shadow track for each total solar eclipse. When you solve each pair of equations for X and Y, you can recover the longitude and latitude of the intersections points, shown in the last two columns. If you are within 50 kilometers of these points, you would have seen two total solar eclipses separated by a few years. The closest pairing is for the 7 years between the 2017 and the upcoming 2024 eclipse pair, which will happen over Cedar Lake in Illinois.

Problem: Calculate the intersection point for each track equation with the one for 2017 and find the geographic longitude and latitude of this point.

Date	Fitting function	Crossing Point	
		Longitude (° ' " West)	**Latitude** (° ' " North)
August 21, 2017	$y = -0.005792x^2 + 1.4425x - 44.968$		
March 27, 1503	$y = -0.00441x^2 + 0.8998x + 0.1113$	119 19 15	44 41 20
July 20, 1506	$y = -0.003139x^2 + 0.2726x + 34.776$	84 15 55	35 27 28
February 3, 1562	$y = 0.029785x^2 - 7.6792x + 537.25$	119 52 07	44 43 12
July 21, 1618	$y = -0.009871x^2 + 2.6021x - 124.7$	116 29 56	44 28 21
October 23, 1623	$y = 0.005971x^2 - 1.1966x + 92.538$	82 25 29	34 34 47
April 10, 1679	$y = -0.00392x^2 + 0.3425x + 49.689$	104 42 40	42 34 18
May 22, 1724	$y = -0.00729x^2 + 0.8719x + 32.78$	106 29 13	42 57 40
June 24, 1778	$y = -0.005793x^2 + 0.5098x + 31.2$	81 39 24	34 12 06
June 16, 1806	$y = -0.00508x^2 + 0.6976x + 18.627$	93 46 48	39 22 15
November 30, 1834	$y = 0.01238x^2 - 1.8789x + 103.4$	105 03 57	42 39 09
July 29, 1878	$y = 0.000486x^2 + 0.7316x - 42.9$	110 14 55	43 39 55
January 1, 1889	$y = 0.00328x^2 - 1.1774x + 134.37$	111 30 27	43 51 51
May 28, 1900	$y = -0.00147x^2 - 0.2253x + 62.33$	81 34 59	34 9 54
June 8, 1918	$y = -0.00192x^2 + 0.8297x - 26.41$	117 27 36	44 33 24
March 7, 1970	$y = -0.0142\ x^2 + 1.394x + 12.72$	79 59 51	33 21 43
April 8, 2024	$y = -0.012722x^2 + 1.6454x - 7.8473$	89 16 38	37 38 58
May 11, 2078	$y = -0.0119x^2 + 1.5792x - 15.458$	81 35 35	34 10 13

Example: March 27, 1503 and August 21, 2017:

2017: $y = -0.005792x^2 + 1.4425x - 44.968$
1503: $y = -0.00441x^2 + 0.8998x + 0.1113$

$-0.005792x^2 + 1.4425x - 44.968 = -0.00441x^2 + 0.8998x + 0.1113$
$0 = (-0.00441+0.005792)x^2 + (0.8998-1.4425)x + (0.1113+44.968)$

$0 = 0.001382\ x^2 - 0.5427\ x + 45.0793$

Solve this equation for its two roots using the quadratic formula where

a = 0.001382,
b=-0.5427,
c=45.0793, then

$$X = \frac{0.5427 +/- [\ 0.5427^2 - 4(0.001382 \times 45.0793)]^{1/2}}{2\ (0.001382)}$$

$$X = \frac{0.5427 +/- 0.212896}{0.002764}$$

X = 196.346 +/- 77.0246

X1 = +273.371 so this is 273.371 west longitude, which is not over the continental USA
X2 = +119.321 so this is 119.321 west longitude, which is over the continental USA.

X2 is the real intersection solution.

We now insert 119.321 in one of the quadratic equations for y = latitude to get the corresponding latitude.

$Y = -0.005792\ (119.321)^2 + 1.4425(119.321) - 44.968$
$Y = -82.4636 + 172.1205 - 44.968$
$Y = 44.6889$

The intersection point for the 2017 and 1503 eclipses is at (119.321 West, +44.689 North) or (119° 19' 15" west , +44° 41' 20" North)

The vast majority of these locations are far removed from cities and towns, but in all cases a short half-day hike would suffice to reach most of these locations. In advance of visiting these places, you can explore them with GOOGLE Earth. Here is a sampling of the scenery you may find.

The crossing points in Oregon and Idaho for 1503, 1562, 1618, 1889 and 1918

Crossing points in Wyoming for 1724, 1834 and 1878

Two crossing points in Missouri and Tennessee for 1506, 1806 and 2024.

Remaining crossing points in South Carolina for 1623, 1778, 1900, 1970 and 2078, which include the Colonial Era, and the Revolutionary War.


Kentucky
North Carolina
ZD
Blue Ridge Mountains
B
1506
1623
1900
1778
2078
1970
S
1834b
Tennessee
Data SIO, NOAA, U.S. Navy, NGA, GEBCO
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