Answer Key for Challenge Problems

Challenge 1 – Working with Geographic Coordinates

Problem1

A) New York City, Statue of Liberty: 74.04465 West 40.6893 North
B) Kennedy Space Center: 80.6817 West, 28.5235 North
C) Golden Gate Bridge: 122 : 28 : 41.9 West 37 : 49 : 7.7 North,
D) Chaco Canyon Ruins : 107 : 57 : 22.0 West 36:05:53 North

Problem 2

A) Latitude 35^o : 28' : 15.5"
B) Longitude 115^o : 15' : 33.2"

Problem 3 - Convert all coordinates to decimal degrees to get Washington (38.905^o, 77.037^o) and Portland (45.544, 122.654). Find the mid-point between the Latitudes: (38.905+45.544)/2 = 42.225 or 42^o : 13' : 30" North. Find the mid-point between the longitudes: (77.037+122.654)/2 = 99.846 or 99^o : 50' : 46" West. The location of the mid-way station is at (42^o : 13' : 30" North , 99^o : 50' : 46" West). This is 35 miles south of Ainsworth, Nebraska!

Problem 4

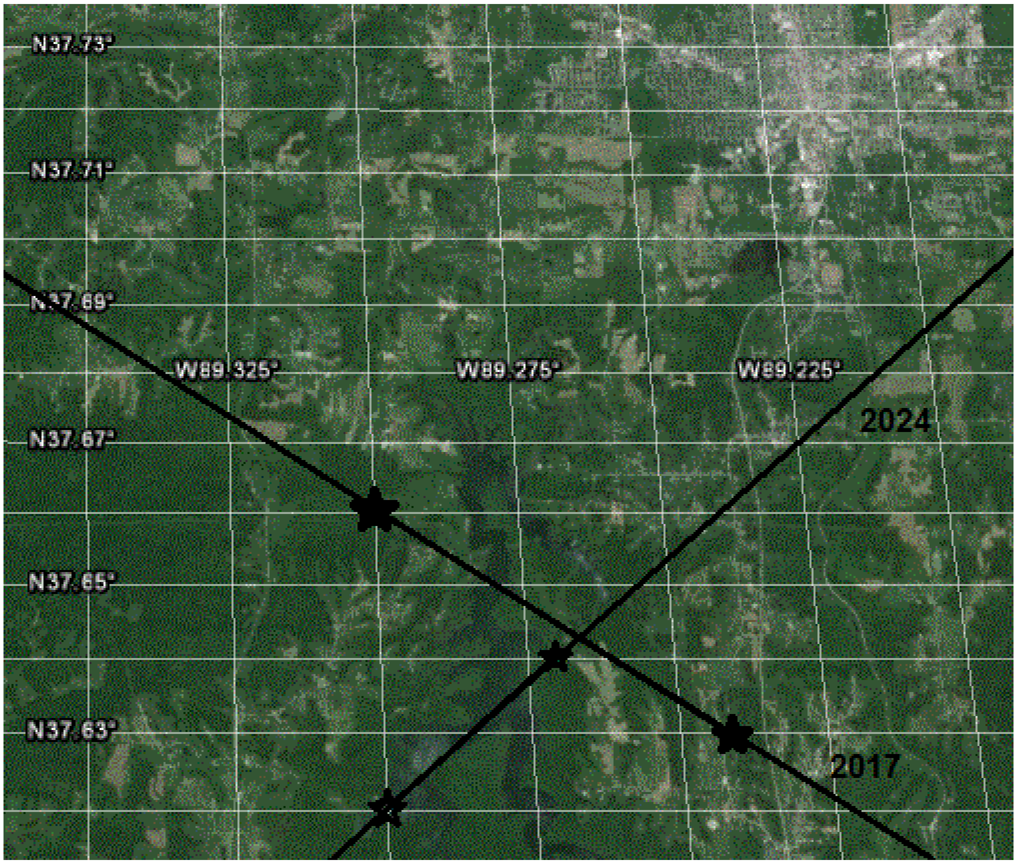
A) Convert to decimal degrees and subtract them: 39.84 – 41.12 = 1.28^o.
B) A full 360-degree great circle has a circumference of 40,000 km, so 1.28^o corresponds to 40000(1.28/360) = 142 kilometers.

Problem 5

A) Convert to decimal degrees and subtract them: 101.71 – 94.64 = 7.07o.
B) 7.07^o corresponds to 7.07 * 86 = 608 kilometers.
C) 7.07 x 38 = 269 kilometers or less than half the distance!

Challenge 2 – X Marks the Spot

Problem 1



Intersection near Longitude 89.27West, 37.64 North.

Problem 2 - About 10 kilometers southwest of Carbondale.

Challenge 3 – X Marks the Spot

Eclipse Date	Eqn for 2017	Eqn for Eclipse	Crossing Point	
			Longitude	**Latitude**
March 27, 1503	y = -0.0647x + 36.8672	y=+0.1557x + 63.3460	-120.1397	44.640
July 20, 1506	y=-0.4728x - 4.4130	y=+0.2608x + 57.4857	-84.3826	35.479
February 3, 1562	y=-0.0660x + 36.7095	y=+0.5038x + 105.2234	-120.229	44.647
July 21, 1618	y=-0.0980x + 32.9103	y = -0.3112x + 8.1419	-116.192	44.304
October 23, 1623	Y=-0.4960x – 6.3555	Y = +0.1808x + 49.3526	-82.306	34.473
April 10, 1679	Y=-0.2165x + 19.8103	Y = +0.4931x + 94.2527	-104.899	42.525

May 22, 1724	Y=-0.1932x + 22.2864	Y = +0.6949x + 116.9709	-106.622	42.880
June 24, 1778	Y=-0.5030x -6.9270	Y = +0.4459x + 70.7603	-81.8694	34.256
June 16, 1806	Y=-0.3496x + 6.5846	Y = +0.2676x + 64.5303	-93.8733	39.406
November 30, 1834	Y=-0.2231x + 19.1221	Y = -0.7248x – 33.1709	-104.233	42.378
July 29, 1878	Y=-0.2911x + 11.4790	Y = -0.8411x – 48.905	-109.789	43.446
January 1, 1889	Y=-0.1598x + 25.8940	Y=+0.4636x + 95.5417	-111.717	43.752
May 28, 1900	Y=-0.4978x - 6.503	Y= +0.4699x + 72.5603	-81.6964	34.1694
June 8, 1918	Y=-0.0954x + 33.2274	Y = -0.3598x + 2.4180	-116.48	44.3338
March 7, 1970	Y=-0.5105x – 7.5342	Y = +0.8788x + 103.7412	-80.0959	33.3539
April 8, 2024	Y=-0.4129x + 0.787	Y = +0.616x + 92.6308	-89.2655	37.6433
May 11, 2078	Y=-0.5024x – 6.8741	Y=+0.3681x + 64.1946	-81.6446	34.1432

Challenge 4 – X Marks the Spot with Quadratic Equations

Problem 1

Date	Fitting function	Crossing Point	
		Longitude (° ‘ “ West)	**Latitude** (° ‘ “ North)
August 21, 2017	$y = -0.005792x^2 + 1.4425x – 44.968$	89 16 38	37 38 56
March 27, 1503	$y = -0.00441x^2 + 0.8998x + 0.1113$	119 19 15	44 41 20
July 20, 1506	$y = -0.003139x^2 +0.2726x + 34.776$	84 15 55	35 27 28
February 3, 1562	$y = 0.029785x^2 - 7.6792x + 537.25$	119 52 07	44 43 12
July 21, 1618	$y = -0.009871x^2 + 2.6021x - 124.7$	116 29 56	44 28 21
October 23, 1623	$y = 0.005971x^2 - 1.1966x + 92.538$	82 25 29	34 34 47
April 10, 1679	$y = -0.00392x^2 + 0.3425x + 49.689$	104 42 40	42 34 18

May 22, 1724	$y = -0.00729x^2 + 0.8719x + 32.78$	106 29 13	42 57 40
June 24, 1778	$y = -0.005793x^2 + 0.5098x + 31.2$	81 39 24	34 12 06
June 16, 1806	$y = -0.00508x^2 + 0.6976x + 18.627$	93 46 48	39 22 15
November 30, 1834	$y = 0.01238x^2 - 1.8789x + 103.4$	105 03 57	42 39 09
July 29, 1878	$y = 0.000486x^2 +0.7316x – 42.9$	110 14 55	43 39 55
January 1, 1889	$y = 0.00328x^2 -1.1774x + 134.37$	111 30 27	43 51 51
May 28, 1900	$y = -0.00147x^2 -0.2253x + 62.33$	81 34 59	34 9 54
June 8, 1918	$y = -0.00192x^2 + 0.8297x - 26.41$	117 27 36	44 33 24
March 7, 1970	$y = -0.0142 x^2 +1.394x+ 12.72$	79 59 51	33 21 43
April 8, 2024	$y = -0.012722x^2 + 1.6454x - 7.8473$	89 16 38	37 38 58
May 11, 2078	$y = -0.0119x^2 + 1.5792x - 15.458$	81 35 35	34 10 13

Challenge 5 – Estimating the speed of the lunar shadow

Problem 1 - Speed = 232 km/0.0619 hours = 3750 km/hr

Problem 2 - Convert the times to decimal hours to get Carbondale: 18.3656 hours and Hopkinsville: 18.4342 hours. The time difference is then 0.069 hours. The speed is then Speed = 155 km/0.069 hours = 2450 km/hr

Problem 3 –

	Latitude	Longitude	Time	Distance (km)	Speed (km/h)
Newport	44.8	-124.0	17:16:58	0	
Madras	44.7	-121.1	17:20:41	232	3750
Weiser	44.4	-117.0	17:26:21	328	3480
Idaho Falls	43.8	-111.9	17:34:14	417	3170
Riverton	43.2	-108.2	17:40:24	301	2930
Casper	42.8	-106.3	17:43:51	161	2800
Stapleton	41.5	-100.5	17:55:21	507	2650
St Joseph	39.8	-94.9	18:07:45	512	2480
Columbia	38.8	-92.3	18:13:57	247	2390
Carbondale	37.6	-89.1	18:21:56	313	2360
Hopkinsville	36.9	-87.5	18:26:03	161	2340
Anderson	34.6	-82.6	18:39:12	511	2330
Columbia	33.9	-81.1	18:43:10	156	2370
McClellanville	33.1	-79.5	18:47:30	172	2380

Worked example: Between Weiser and Idaho Falls, the distance is 417 kilometers, and the time difference is (17:34:14 – 17:26:21) = 17.5706 – 17.4392 = 0.1314 hours, so speed = 417 km/0.1314 h = 3173 km/hr, which rounded to three significant figures becomes 3170 km/hr.

Problem 4 - Add up the distances in column 5 to get 4019 kilometers.

Problem 5 - At Newport, totality starts at 17:16:58 and at McClellanville it starts at 18:47:30, a difference in time of 18.7917 – 17.2828 or 1.51 hours. The average speed is then 4019 km/1.51 hours = 2661 km/hour, and rounded to three significant figures this becomes 2660 km/hr.

Challenge 6 – As the crow flies on a spherical planet

Problem 1 - Lat1 = 44.8439 Long1 = 124.051 Lat2= 33.0354 Long2 = 79.4869

Then
cos(X) = sin(44.8439)sin(33.0354) + cos(44.8439)cos(33.0354)cos(124.051-79.4869)
cos(X) = 0.7052 x 0.5452 + 0.7090 x 0.8383 x 0.7125
cos(X) = 0.8079

So X = 36.1^{o} and the distance is then 40000 (36.1/360) = 4,011 kilometers.

Problem 2 - lat1= 38.9 long1 = 77.05 lat2=34.00 long2=81.05

Then
cos(X) = sin(38.9)sin(34.0) + cos(38.9)cos(34.0)cos(81.05-77.05)
cos(X) = 0.628 x 0.559 + 0.778 x 0.829 x 0.998
cos(X) = 0.995

so X = 5.9^{o} and so the distance is 40000 x (5.9/360) = 660 kilometers.

Challenge 7 – Exploring the lunar shadow cone.

Problem 1 - The sun is 151.39 million km from the center of Earth and the moon is 372,027 km from the center of Earth, so the distance from the center of the sun to the center of the moon is 151,390,000 – 372,000 = 151,018,000 km, then

$$\frac{696{,}300}{151{,}018{,}000 + X} = \frac{1737}{X}$$

$$X = \frac{151{,}018{,}000\ (1737)}{696{,}300 - 1737}$$

X = 377,673 km. To 4 significant figures: **X = 377,700 km**

Now the moon is 372,027 km sunwards of the Earth's center (USNO), so the vertex of the shadow cone is 377,700 – 372,027 = 5,673 km (to 4-SF) from the center of Earth opposite the sun or 6378-5673 = 705 km below the surface of Earth!

Problem 2 –

$$\frac{696{,}300}{228{,}000{,}000 + X} = \frac{25}{X}$$

$$X = \frac{228{,}000{,}000\ (25)}{696{,}300 - 25}$$

X = 8,186 km.

The vertex of the shadow cone is located 9300-8186 = 1,114 km from the center of Mars. The radius of Mars is 3,396 km, so the vertex is located 3,396-1,114 = 2,282 km below the surface of Mars.

Challenge 8 – Exploring angular diameter

Problem 1 Angular diameter = 206265 x 1392600/151384000 = **1897 arcseconds**.
Problem 2 Angular diameter = 206265 x 3474/365600 = 1960 arcseconds.
Problem 3 - Angular diameter = 206265 * 3474/377,700 = **1897 arcseconds**.

This is the same as the angular diameter of the sun, and so at the vertex the disk of the moon exactly covers the disk of the sun for a total solar eclipse. At the surface of Earth, however, the angular diameter of the moon is 37 arcseconds smaller than the sun, so the dark lunar disk will not appear to fully cover the sun.

Challenge 9 – Lunar shadow size on Earth's surface

Problem 1 The diameter is about 110 km, so it will take 110 km / (2600 k/h) = 0.042 hours or about 2.5 minutes from start to finish. After that time, the limb of the sun will brighten enormously and darkness will become daylight again. This does not leave much time for photographers to set up their equipment correctly and snap a few pictures!

Problem 2 H = (8186-9300 + 3396) 3396 / 8186 = 946 km !

Because of the scattering of light in the dusty Martian atmosphere, a dark spot with this diameter would not be visible. Also, the diameter of Phobos would appear to be significantly smaller than the sun so there would be no total solar eclipse.

Challenge 10 – Shadow speed and Earth's rotation

Problem 1 The Surface is moving eastward at a speed of 1674 cos(42.8) = 1228 km/hr so the true shadow speed is 2,800 + 1,200 = 4,000 km/hr.

Problem 2

A) Vrot = 1674cos(19) = 1600 km/h. True speed = 2,200 km/h + 1600 km/h = 3,800 km/h. The speed relative to the Concorde, which is sharing Earth's rotation, is just 2,200 km/h – 2,100 km/h = 100 km/h.

B) Total duration of the eclipse was about 250 km / (100 km/h) = 2.5 hours. If you were on the ground, the maximum duration would have only been about 7 minutes! The actual time the astronomers were able to get after meeting up with the shadow and maneuvering to keep pace with it was about 1.5 hours!

Challenge 11 – Modeling Shadow Speed, Diameter and Duration along the Path of Totality

Problem 1

A) Use the Quadratic Formula to determine the roots of the equation.
We see, at once that the discriminant $\mathbf{b^2}$**-4ac** is negative so there are no crossing points where V is less than zero.

B) This is a parabolic curve that opens upwards and has a vertex at L = -b/2a = -179/(2*1.06) = - 84.4, which is longitude 84.4 West. The point on the path of totality with this longitude has a latitude of 35.53 North and is located 90 kilometers northeast of Chattanooga, Tennessee. The speed there is found from the formula for V as 1.06(-84.4)^2 +179(-84.4) + 9900 = 2,400 km/hr.

Problem 2

A) This is a parabola that opens downwards so its vertex is a maximum value for D(L). The discriminant b^2-4ac is negative so there are no longitudes for which the shadow diameter is zero.

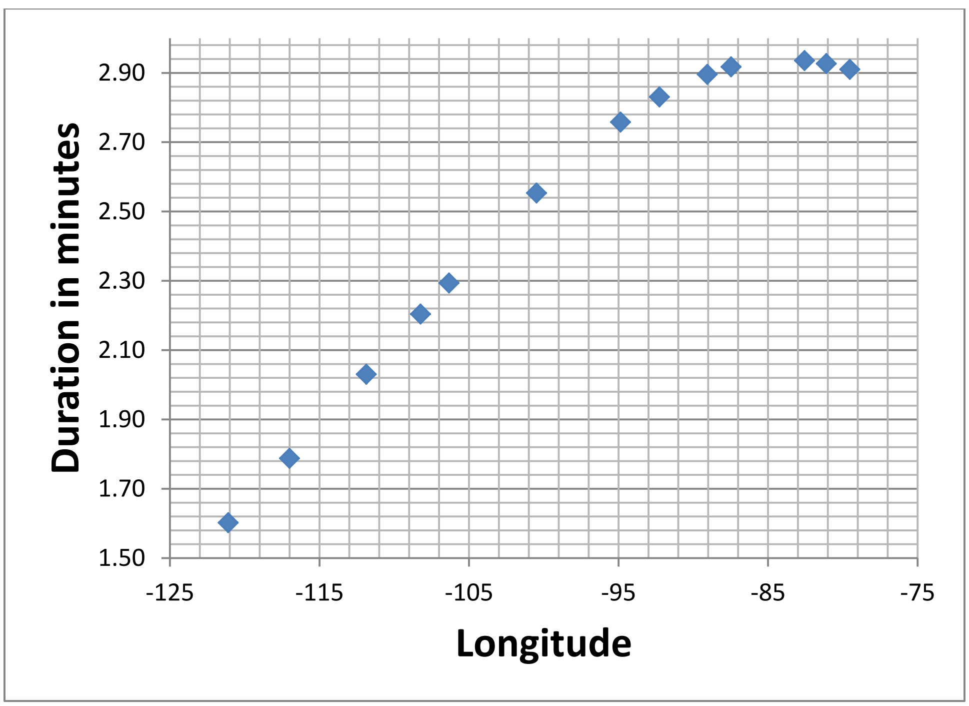
B) The vertex is located at L= -b/2a = -79.8 and has a maximum value of 114.7 kilometers.

Problem 3

A) Time = distance/speed so

$$T(L) = \frac{-0.0084L^2 - 1.3423L + 61.245}{1.06L^2 + 179L + 9900} \times (60 \text{ minutes/hr})$$

B) See below



The eclipse will last about 2 minutes and 54 seconds near a longitude of -85. This is a location near Crossville, Tennessee.

Note: The actual location of maximum duration is 240 km northwest near Hopkinsville, IL (longitude -87.6) where the maximum duration is 2 minutes 40 seconds. The shape of the shadow is elliptical and so the actual function D(L) is more complicated than the one used, which overestimates the size of the shadow leading to longer eclipse times. A 10% change in the shadow diameter reduces the time by about 15 seconds, and moves the location of maximum eclipse further west towards Hopkinsville.

Challenge 12 - A Physics-based Model for Estimating the Shadow Ground Speed

Problem 1

Location	Latitude (L)	Actual speed	Predicted	Difference
Madras, OR	44.7	3,750	2,380	1,370
Casper, WY	42.8	2,800	2,340	460
Carbondale, IL	37.6	2,350	2,240	110
McClellanville, SC	33.0	2,400	2,160	240

Answer B) The farther you are from the Atlantic Coast along the path, the more the model seems to underestimate the shadow speed. The differences vary from a high of 37% on the Pacific Coast to a low of 5% towards the Atlantic Coast. As a model, it matches the data to better than 15% east of Casper, Wyoming.

Problem 2

Location	Latitude	□	Obs. Speed	Predicted	Difference
Madras, OR	44.7	34°	3,750	2,870	880
Casper, WY	42.8	23°	2,800	2,540	260
Carbondale, IL	37.6	9°	2,350	2,270	80
McClellanville, SC	33.0	0°	2,400	2,160	240

We see that the differences now range from 23% near Madras, to a maximum of 9% east of Casper. This is a significant improvement, but we still have the trend that points farther to the west of McClellanville are underestimating the speed.

Problem 3

Location	Latitude	□	Elev	Obs	Predicted	Difference
Madras, OR	44.7	34°	42°	3,750	4,290	-540
Casper, WY	42.8	23°	54°	2,800	3,140	-340
Carbondale, IL	37.6	9°	64°	2,350	2,610	-260
McClellanville, SC	33.0	0°	61°	2,400	2,470	70

The predictions are now larger than the actual measurements and vary from 14% in Madras to 3% in McClellanville.

Challenge 13 – The last total solar eclipse on Earth

Problem 1

Answer: A) $\theta = 206265 \times (1{,}392{,}000 / 149{,}000{,}000) = 1927$ arcseconds

B) 1927 arcseconds x (1 degree/3600 arcseconds) = 0.5 degrees

Problem 2 $0.0073\,(4.5)^2 - 0.028\,(4.5) + 0.98 = 1.00.$

Problem 3

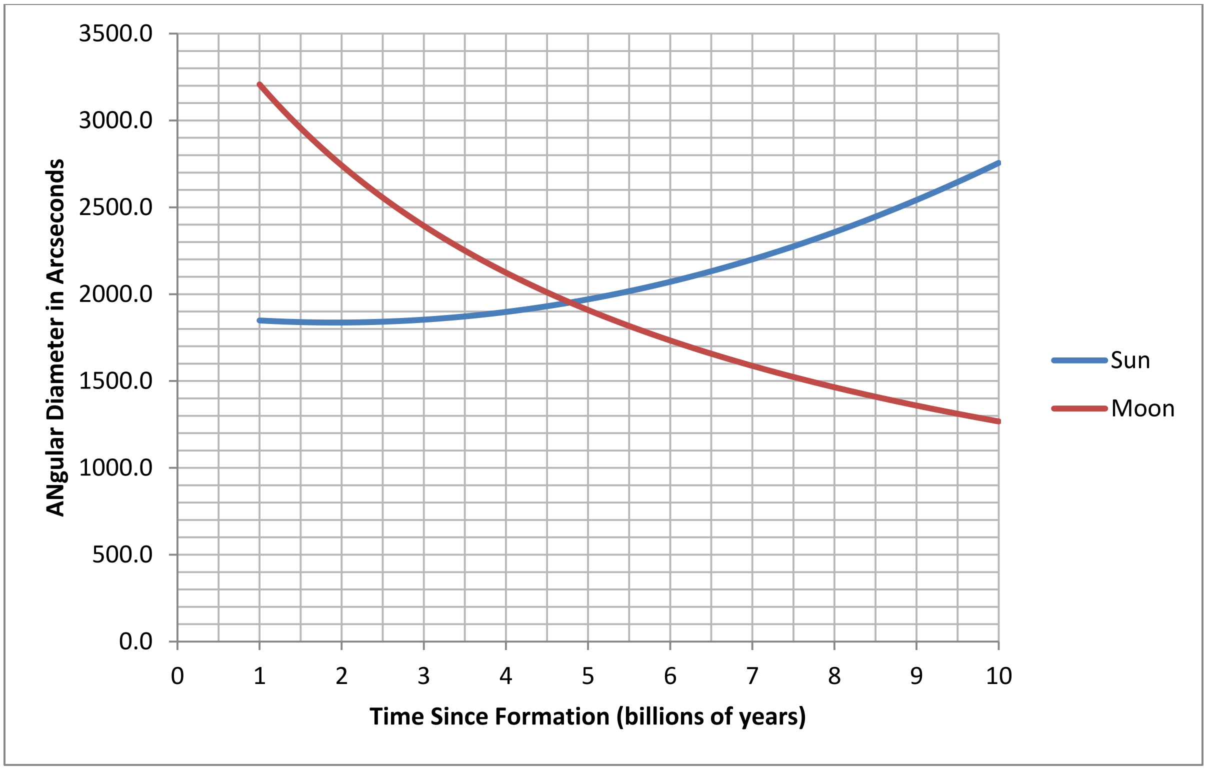
$$\theta(T) = 206265 \; \frac{1392000\,(0.0073\,T^2 - 0.028\,T + 0.98)}{\text{Distance to the sun in kilometers}}$$

Problem 4 - In 1 billion years, the distance traveled will be 3.8 cm/yr x 1 billion years x (1 km/100000 cm) = 38,000 km, so the rate of increase is 38,000 km/billion years. The current distance (T=4.5) is 356,400 km, so the formula would be D = 38,000 T +185,400 km where T is the time in billions of years.

Problem 5 –

$$\theta(T) = 206265 \; \frac{3{,}474}{38000\,T + 185{,}400}$$

Problem 6



Answer: The two functions cross at an age of about 4.75 billion years, which is 250 million years from today!

Challenge 14 - The View from Mars: A matter of time

Problem 1 - A Martian day is 24h 39.6 minutes long by Earth time-keeping, which in decimal value is equal to 24.66 so that means that a Martian hour is equal to 24.66/24 = 1.02749 Earth hours, and also that a Martian minute equals 1.02749 Earth minutes. In other words, a Martian minute equals about 62 Earth seconds.

Problem 2 - (August 30 – August 6) = 24 Earth Days. Since 1 Sol equals 1.02749 Earth days, that means 24 Earth days is the same as 24/1.02749 = Sol 23 on the Curiosity calendar.

Problem 3 - First calculate the number of Earth days between August 6, 2012 and August 21, 2017 including leap-days. You can use an online calculator to compute this, for instance, *http://www.timeanddate.com/date/duration.html*. The answer is 1842 Earth days. Next convert Earth days to Sols: Sols = 1842/1.02749 = 1792, so the Eclipse will happen on Sol 1792 at the Curiosity lander.

Problem 4 - First calculate the number of Earth days between August 6, 2012 and August 21, 2017 including leap-days. You can use an online calculator to compute this, for instance, *http://www.timeanddate.com/date/duration.html*. The answer is 1842 Earth days. Next convert Earth days to Sols: Sols = 1842/1.02749 = 1792, so the Eclipse will happen on Sol 1792 at the Curiosity lander.

<u>Challenge 15 - Angular Size and the Earth-Moon System</u>

Problem 1 It would appear in the sky to have an angular diameter of 1.9 degrees.

Problem 2 The moon has a diameter of only 0.5 degrees, so Earth is about 4 times bigger in the sky as viewed from our moon!

Problem 3

$$\frac{\text{Object 1}}{\text{Object 2}} = \frac{r}{R} \quad \frac{X + D}{X}$$

Problem 4

$$\frac{\text{Moon}}{\text{Earth}} = \frac{1750}{6378} \quad \frac{6.2 \text{ million} + 384{,}000}{6.2 \text{ million}} = 0.27 \times 1.06 = 0.28$$

Problem 5

$$\text{Earth:} \quad 57.29 \; \frac{12{,}700}{200{,}000} = 3.6 \text{ degrees}$$

$$\text{Moon:} \quad 57.29 \; \frac{3{,}500}{8{,}000} = 25 \text{ degrees}$$

The ratio of their diameters was about 25/3.6 or about 7 to 1. This matches the apparent diameters seen in the photo.

Problem 6

A) X = 150 million – 40 million = 110 million from sun to Venus

$$\frac{\text{Venus}}{\text{Sun}} = \frac{12{,}700}{1.4 \text{ million}} \; \frac{40 \text{ million} + 110 \text{ million}}{110 \text{ million}} = 0.0124$$

B) D = 225 million – 150 million = 75 million km from Earth to Mars

$$\frac{\text{Earth}}{\text{Sun}} = \frac{12{,}700}{1.4 \text{ million}} \; \frac{150 \text{ million} + 75 \text{ million}}{75 \text{ million}} = 0.027$$

So from Mars, Earth will appear as a black dot on the face of the sun that is more than twice as big as Venus appeared during the 2012 Transit of Venus!

Problem 7

$$\frac{1}{2.9} = \frac{1750}{6378} \; \frac{1.6 \text{ million} + D}{1.6 \text{ million}}$$

$$1.24 = 1 + D/ 1.6 \text{ million}$$

$$D = 384{,}000 \text{ km.}$$

Problem 8

The apparent diameter ratio of Phobos to the sun is 12:20 so

$$\frac{12}{20} = \frac{25}{1.4 \text{ million}} \; \frac{X + 225 \text{ million}}{X}$$

X = 6,700 kilometers.