## X Marks the Spot - Linear Equations

One interesting feature of the August 21, 2017 total solar eclipse across the continental United States is that it crosses the path of other, older eclipses that have occurred since 1503 when North America was first discovered. Here are what the eclipse paths look like for previous eclipses.
The black track is for August 21, 2017.


## Fun with Linear Equations!

For the mathematically-inclined, this table gives the linear portions of each eclipse track as two equations. When you solve each pair for X and Y , you can recover the longitude and latitude of the intersections points, shown in the last two columns. If you are within 50 kilometers of these points, you would have seen two total solar eclipses separated by a few years. The closest pairing is for the 7 years between the 2017 and the upcoming 2024 eclipse pair, which will happen over Cedar Lake in Illinois.

Problem: Calculate the intersection point for each linear equation with the one for 2017 and find the geographic longitude and latitude of this point.

| Eclipse Date | Eqn for 2017 | Eqn for Eclipse | Crossing Point |  |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  | Longitude | Latitude |
| March 27, 1503 | $\mathrm{y}=-0.0647 \mathrm{x}+36.8672$ | $\mathrm{y}=+0.1557 \mathrm{x}+63.3460$ | -120.1397 | 44.640 |
| July 20, 1506 | $\mathrm{y}=-0.4728 \mathrm{x}-4.4130$ | $\mathrm{y}=+0.2608 \mathrm{x}+57.4857$ | -84.3826 | 35.479 |
| February 3, 1562 | $\mathrm{y}=-0.0660 \mathrm{x}+36.7095$ | $\mathrm{y}=+0.5038 \mathrm{x}+105.2234$ | -120.229 | 44.647 |
| July 21, 1618 | $\mathrm{y}=-0.0980 \mathrm{x}+32.9103$ | $\mathrm{y}=-0.3112 \mathrm{x}+8.1419$ | -116.192 | 44.304 |
| October 23, 1623 | $\mathrm{Y}=-0.4960 \mathrm{x}-6.3555$ | $\mathrm{Y}=+0.1808 \mathrm{x}+49.3526$ | -82.306 | 34.473 |
| April 10, 1679 | $\mathrm{Y}=-0.2165 \mathrm{x}+19.8103$ | $\mathrm{Y}=+0.4931 \mathrm{x}+94.2527$ | -104.899 | 42.525 |
| May 22, 1724 | $\mathrm{Y}=-0.1932 \mathrm{x}+22.2864$ | $\mathrm{Y}=+0.6949 \mathrm{x}+116.9709$ | -106.622 | 42.880 |
| June 24, 1778 | $\mathrm{Y}=-0.5030 \mathrm{x}-6.9270$ | $\mathrm{Y}=+0.4459 \mathrm{x}+70.7603$ | -81.8694 | 34.256 |
| June 16, 1806 | $\mathrm{Y}=-0.3496 \mathrm{x}+6.5846$ | $\mathrm{Y}=+0.2676 \mathrm{x}+64.5303$ | -93.8733 | 39.406 |
| November 30, 1834 | $\mathrm{Y}=-0.2231 \mathrm{x}+19.1221$ | $\mathrm{Y}=-0.7248 \mathrm{x}-33.1709$ | -104.233 | 42.378 |
| July 29, 1878 | $\mathrm{Y}=-0.2911 \mathrm{x}+11.4790$ | $\mathrm{Y}=-0.8411 \mathrm{x}-48.905$ | -109.789 | 43.446 |
| January 1, 1889 | $\mathrm{Y}=-0.1598 \mathrm{x}+25.8940$ | $\mathrm{Y}=+0.4636 \mathrm{x}+95.5417$ | -111.717 | 43.752 |
| May 28, 1900 | $\mathrm{Y}=-0.4978 \mathrm{x}-6.503$ | $\mathrm{Y}=+0.4699 \mathrm{x}+72.5603$ | -81.6964 | 34.1694 |
| June 8, 1918 | $\mathrm{Y}=-0.0954 \mathrm{x}+33.2274$ | $\mathrm{Y}=-0.3598 \mathrm{x}+2.4180$ | -116.48 | 44.3338 |
| March 7, 1970 | $\mathrm{Y}=-0.5105 \mathrm{x}-7.5342$ | $\mathrm{Y}=+0.8788 \mathrm{x}+103.7412$ | -80.0959 | 33.3539 |
| April 8, 2024 | $\mathrm{Y}=-0.4129 \mathrm{x}+0.787$ | $\mathrm{Y}=+0.616 \mathrm{x}+92.6308$ | -89.2655 | 37.6433 |
| May 11, 2078 | $\mathrm{Y}=-0.5024 \mathrm{x}-6.8741$ | $\mathrm{Y}=+0.3681 \mathrm{x}+64.1946$ | -81.6446 | 34.1432 |

## Example:

March 27, 1503 and August 21, 2017:

$$
\begin{aligned}
& \text { 2017: } \quad \mathrm{y}=-0.0647 \mathrm{x}+36.8672 \\
& \text { 1503: } \mathrm{y}=+0.1557 \mathrm{x}+63.3460 \\
& -0.0647 \mathrm{x}+36.8672=+0.1557 \mathrm{x}+63.3460 \\
& 36.8672-63.3460=0.1557 \mathrm{x}+0.0647 \mathrm{x} \\
& -26.4788=+0.2204 \mathrm{x} \\
& -26.4788 / 0.2204=\mathrm{x} \\
& -120.1397=\mathrm{x}
\end{aligned}
$$

So the point is at west longitude $120.1397^{\circ}$
The latitude is found by substituting -120.1397 into either equation for ' $y$ '.
$\mathrm{Y}=-0.0647(-120.1397)+36.8672$
$\mathrm{Y}=7.7730+36.867$
$\mathrm{Y}=44.64^{\circ}$ north latitude.
The vast majority of these locations are far removed from cities and towns, but in all cases a short half-day hike would suffice to reach most of these locations. In advance of visiting these places, you can explore them with GOOGLE Earth. Here is a sampling of the scenery you may find.

The crossing points in Oregon and Idaho for 1503, 1562, 1618, 1889 and 1918


Crossing points in Wyoming for 1724, 1834 and 1878


Two crossing points in Missouri and Tennessee for 1506, 1806 and 2024.


Remaining crossing points in South Carolina for 1623, 1778, 1900, 1970 and 2078, which include the Colonial Era, and the Revolutionary War.


